Orbital Angular Momentum: Interference, Entangelement, and Precision Measurement

Anand Kumar Jha* and Robert W. Boyd

The Institute of Optics, University of Rochester, Rochester, NY

Girish S. Agarwal

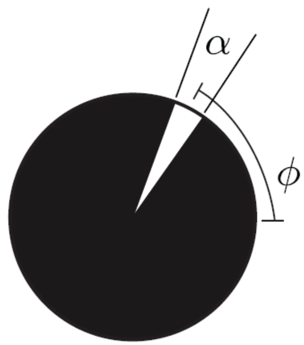
Department of Physics, Oklahoma State University, Stillwater, Oklahoma

Physics of Quantum Electronics (PQE)
Snowbird, Utah, January 2-6, 2011

*now at Intel Corporation, Hillsboro, OR

Orbital Angular Momentum: Basics

Angular position



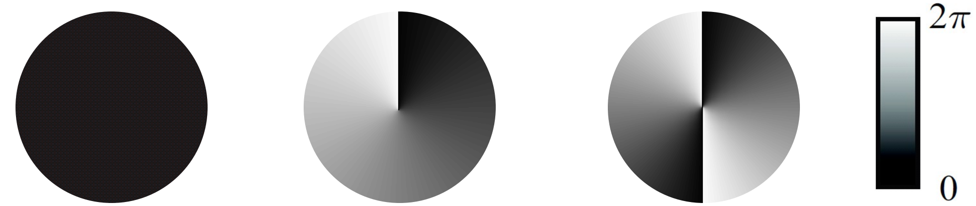
$$A_l = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} d\phi \Psi(\phi) \exp(-il\phi)$$

$$\Psi(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{+\infty} A_l \exp(il\phi)$$

Barnett and Pegg, PRA **41**, 3427 (1990)
Franke-Arnold et al., New J. Phys. **6**, 103 (2004)
Forbes, Alonso, and Siegman J. Phys. A **36**, 707 (2003)

Laguerre-Gauss basis LG_p^l with p=0

$$\mathbf{A} = \hat{x} u(\rho, z) e^{-ikz} e^{il\phi}$$

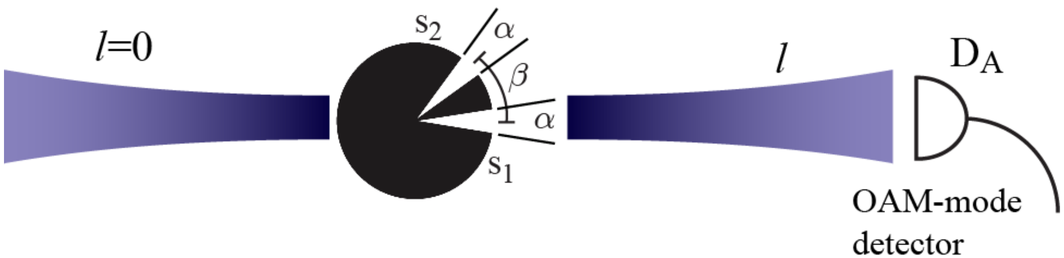


***l*=0** ***l*=1** ***l*=2**

$$\frac{J_z}{W} = \frac{\iint \rho d\rho d\phi (\boldsymbol{\rho} \times \langle \mathbf{E} \times \mathbf{B} \rangle)_z}{c \iint \rho d\rho d\phi \langle \mathbf{E} \times \mathbf{B} \rangle_z} = \frac{\hbar l}{\hbar \omega}$$

Allen et al., PRA **45**, 8185 (1992)

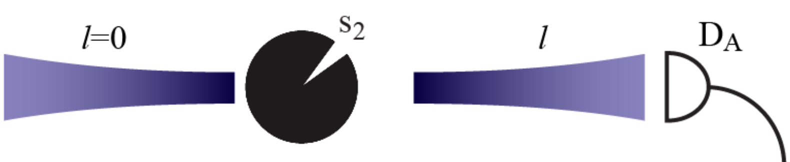
Orbital Angular Momentum: Interference



1

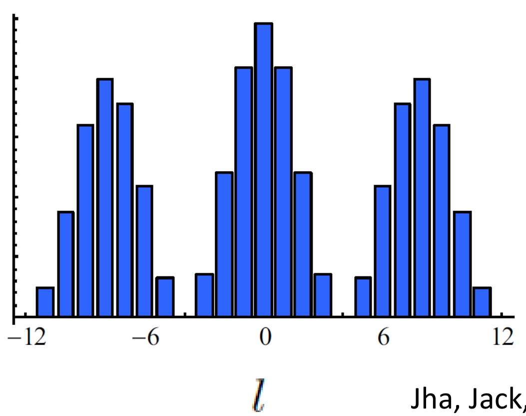
$$\psi_{1l} = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} d\phi \Psi_1(\phi) e^{-il\phi}$$
$$= \frac{\alpha}{\sqrt{2\pi}} \text{sinc}\left(\frac{l\alpha}{2}\right)$$

2



$$\psi_{2l} = \frac{\alpha}{\sqrt{2\pi}} \text{sinc}\left(\frac{l\alpha}{2}\right) e^{-il\beta}$$

$\alpha = \pi/10$
$\beta = \pi/4$



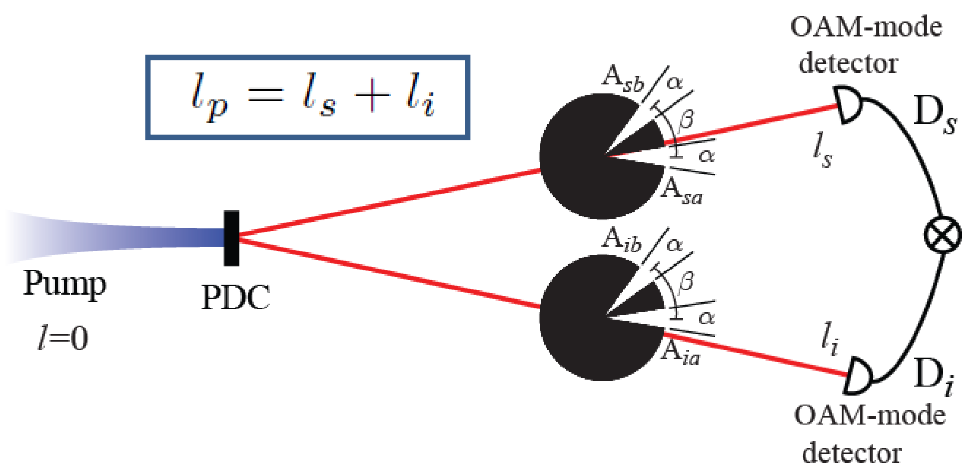
OAM-mode distribution:

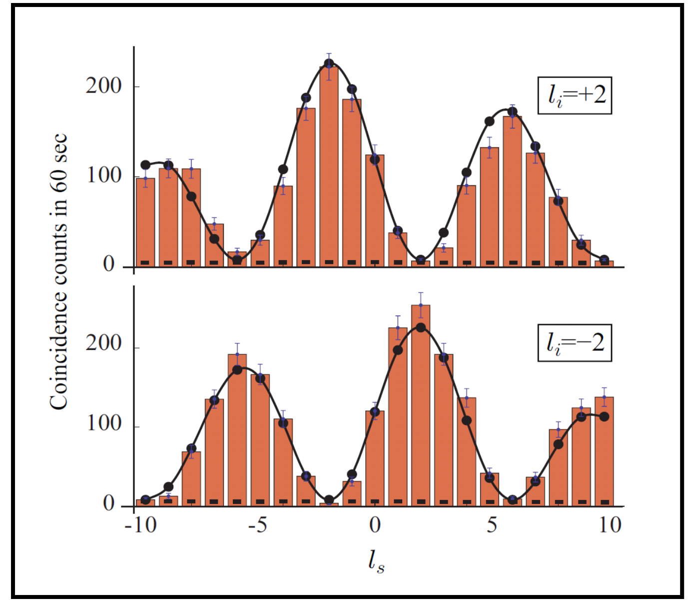
$$I_A = C\frac{\alpha^2}{\pi}\text{sinc}^2\left(\frac{l\alpha}{2}\right)\left[1 + \cos\left(l\beta\right)\right]$$

E. Yao et al., Opt. Express **14**, 13089 (2006)
Jha, Jack, Yao, Leach, Boyd, Buller, Barnett, Franke-Arnold, Padgett, PRA **78**, 043810 (2008)

Orbital Angular Momentum Entanglement

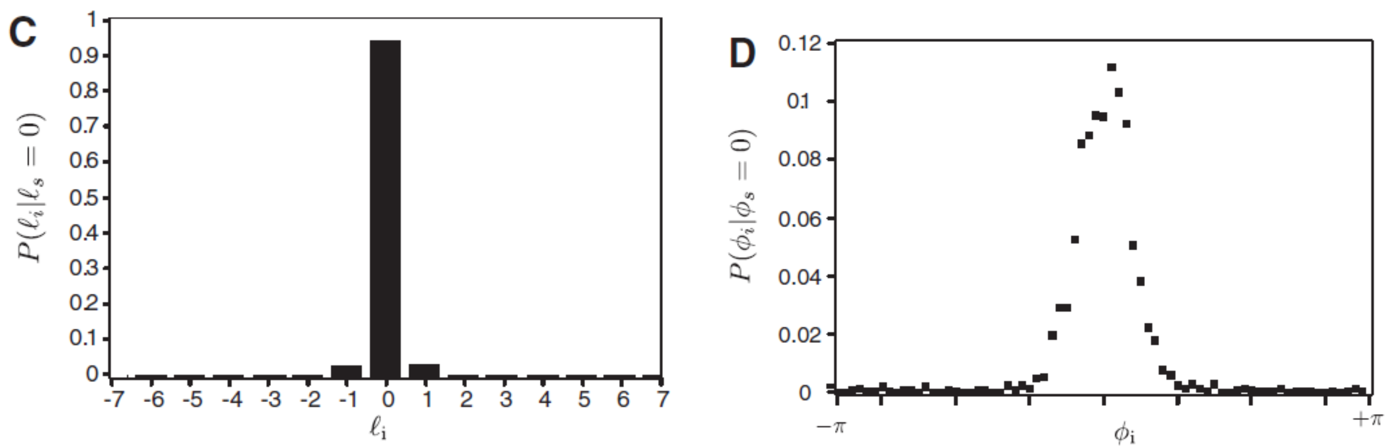
1. Angular two-photon interference





Jha, Leach, Jack, Franke-Arnold, Barnett, Boyd, and Padgett, PRL **104,** 010501 (2010)

2. EPR Correlations

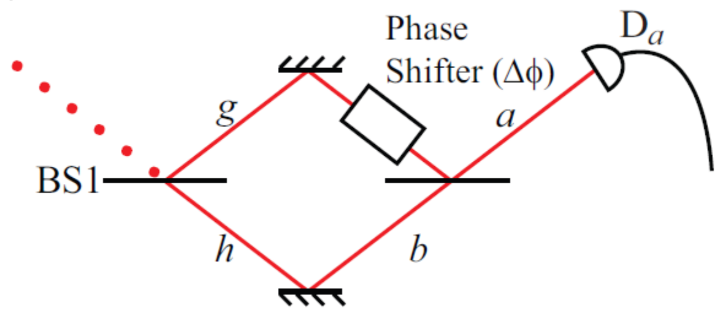


Leach, Jack, Romero, Jha, Yao, Franke-Arnold, Ireland, Barnett, Boyd, and Padgett, Science **329,** 662 (2010)

NOON State & Precision phase measurement (overview)

Given N photons, how precisely can the optical phase be measured?

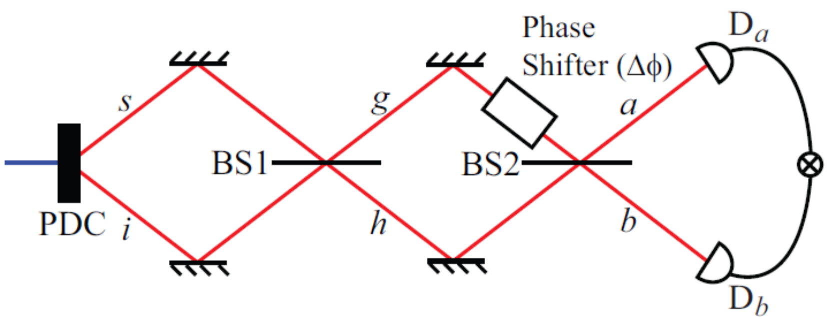
(i) N-single photons



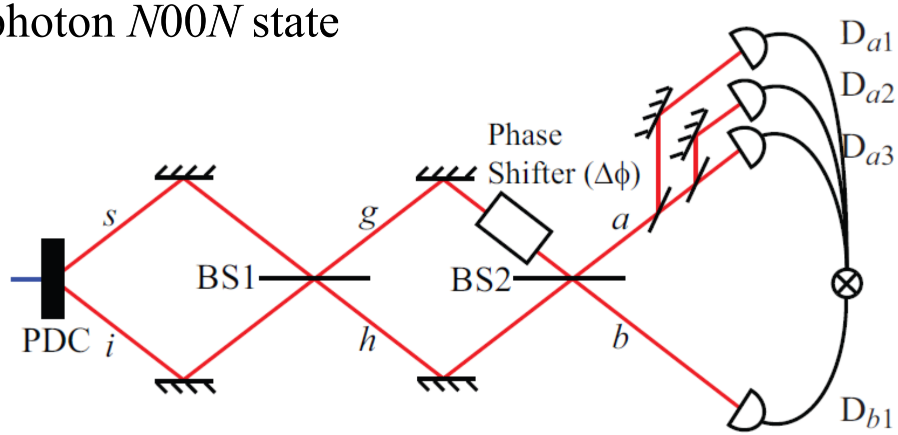
Intensity ~ $\sin\Delta\phi$

Sensitivity ~ $\dfrac{\langle \Delta \hat{A}_2 \rangle}{|\partial \langle \hat{A}_2 \rangle / \partial \Delta\phi|} = \dfrac{1}{\sqrt{N}}$

(ii) 2-photon *N*00*N* state



(iii) 4-photon *N*00*N* state

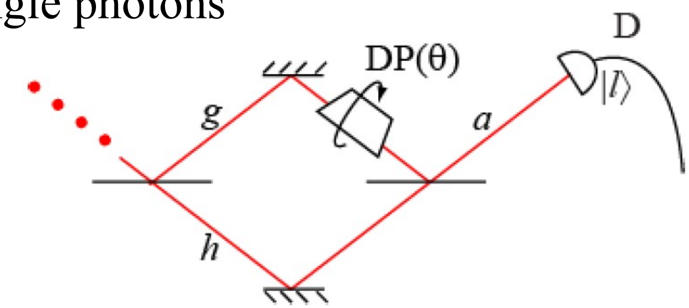


Intensity ~ $\sin N\Delta\phi$

Sensitivity ~ $\dfrac{1}{N}$

NOON State & Precision angle measurement (overview)

(i) N-single photons

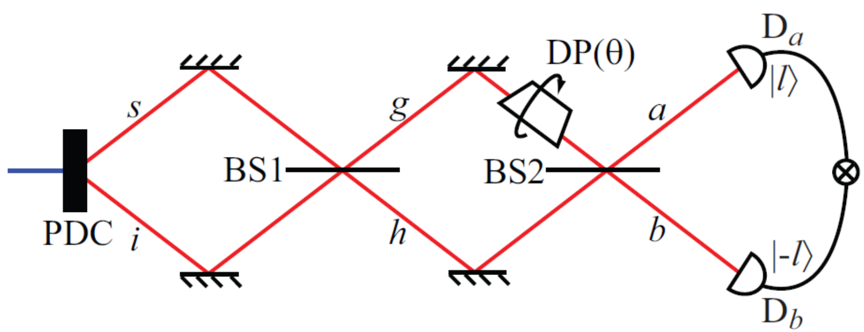


Given N photons with OAM $l\hbar$ per photon, how precisely can angular displacements be measured?

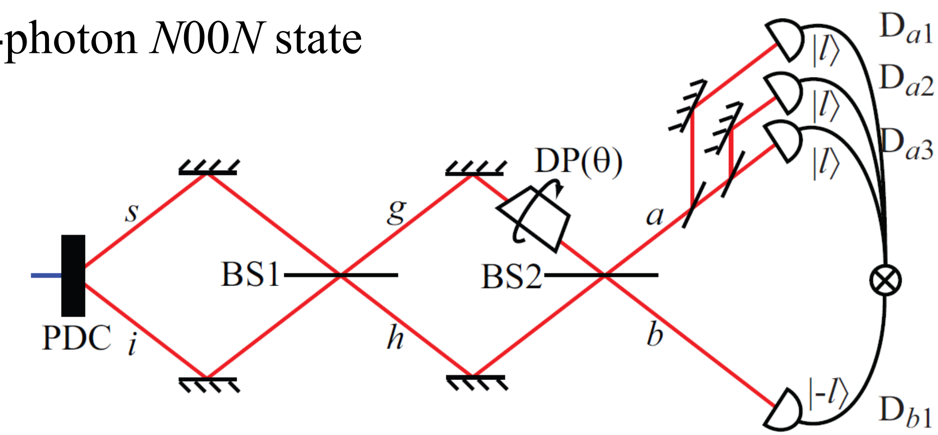
Intensity ~ $\sin 2l\theta$

Sensitivity ~ $\dfrac{\langle \Delta \hat{A}_2 \rangle}{|\partial \langle \hat{A}_2 \rangle / \partial \theta|} = \dfrac{1}{2\sqrt{N}l}$

(ii) 2-photon *N*00*N* state



(iii) 4-photon *N*00*N* state

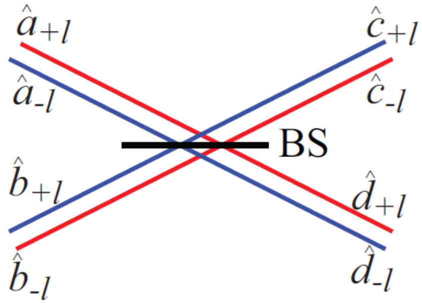


Intensity ~ $\sin 2Nl\theta$

Sensitivity ~ $\dfrac{1}{2Nl}$

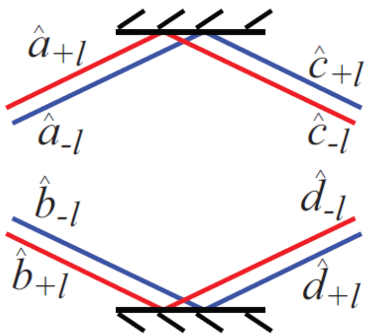
Transformation properties of OAM modes

Beam Splitter



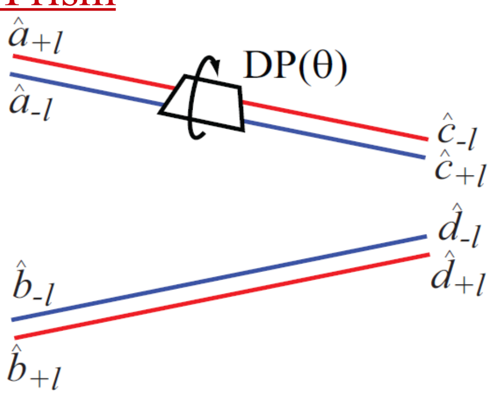
$$\begin{pmatrix} \hat{c}_{+l} \\ \hat{c}_{-l} \\ \hat{d}_{+l} \\ \hat{d}_{-l} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & i & 1 & 0 \\ i & 0 & 0 & 1 \\ 1 & 0 & 0 & i \\ 0 & 1 & i & 0 \end{pmatrix} \begin{pmatrix} \hat{a}_{+l} \\ \hat{a}_{-l} \\ \hat{b}_{+l} \\ \hat{b}_{-l} \end{pmatrix} = M_{\rm bs} \begin{pmatrix} \hat{a}_{+l} \\ \hat{a}_{-l} \\ \hat{b}_{+l} \\ \hat{b}_{-l} \end{pmatrix}$$

Mirror



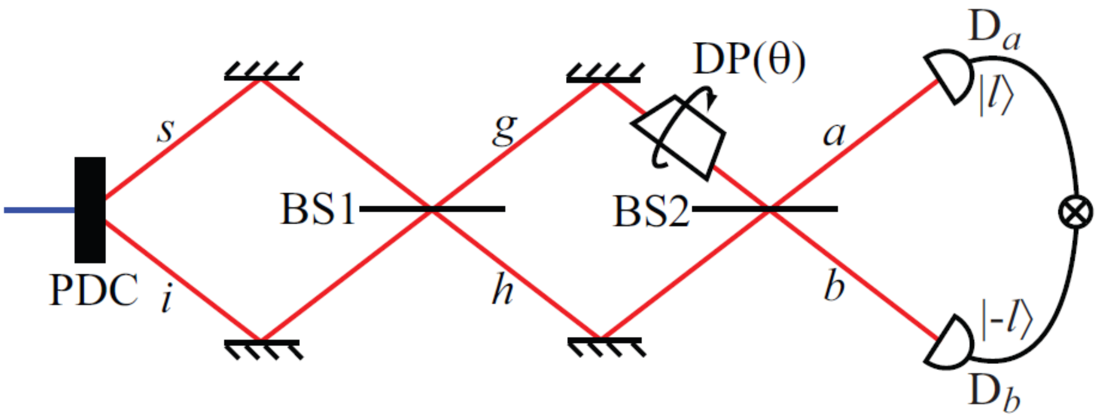
$$M_{\rm mir} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

Dove Prism



$$M_{\rm dp}(\theta) = \begin{pmatrix} 0 & -e^{2il\theta} & 0 & 0 \\ -e^{-2il\theta} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Transformation properties of OAM modes



$$O = M_{\text{bs}} M_{\text{dp}}(\theta) M_{\text{mir}} M_{\text{bs}} M_{\text{mir}} I = MI$$

$$I = M^{-1} O = M^{\dagger} O$$

$$I^{\dagger} = O^{\dagger} M$$

$$O = \begin{pmatrix} \hat{a}_{+l} \\ \hat{a}_{-l} \\ \hat{b}_{+l} \\ \hat{b}_{-l} \end{pmatrix} \quad \text{and} \quad I = \begin{pmatrix} \hat{s}_{+l} \\ \hat{s}_{-l} \\ \hat{i}_{+l} \\ \hat{i}_{-l} \end{pmatrix}$$

$$\boxed{\begin{aligned} \hat{s}_{+}^{\dagger} &= k_1 \hat{a}_{+}^{\dagger} + i k_2 b_{-}^{\dagger}, & \hat{s}_{-}^{\dagger} &= k_1 \hat{a}_{-}^{\dagger} + i k_2 b_{+}^{\dagger} \\ \hat{i}_{+}^{\dagger} &= i k_4 \hat{a}_{-}^{\dagger} + k_3 b_{+}^{\dagger}, & \hat{i}_{-}^{\dagger} &= i k_4 \hat{a}_{+}^{\dagger} + k_3 b_{-}^{\dagger} \end{aligned}}$$

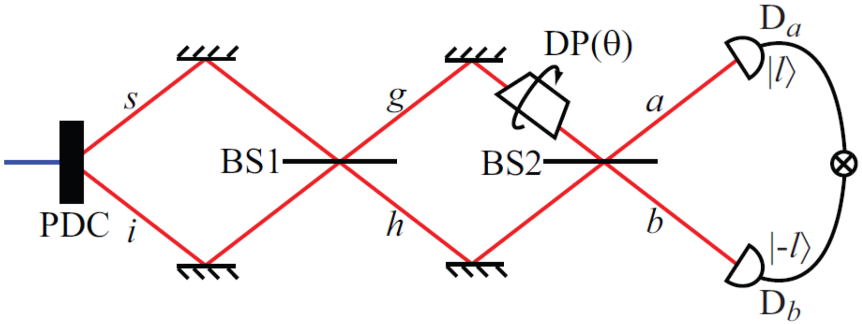
$$k_2 = k_4^{*} = \tfrac{1}{2}(-1 + e^{2il\theta})$$

$$k_1 = k_3^{*} = \tfrac{1}{2}(-1 - e^{2il\theta})$$

Super-sensitive measurement with two-photon entangled state

Two-photon state:

$$|\psi_2\rangle = \sum_l \sqrt{P_l}|l\rangle_s|-l\rangle_i$$



$$|\psi_2^l\rangle = \sqrt{\frac{1}{2}}\left[|1\rangle_{s_{+l}}|1\rangle_{i_{-l}} + |1\rangle_{s_{-l}}|1\rangle_{i_{+l}}\right] = \sqrt{\frac{1}{2}}\left[\hat{s}^\dagger_{+l}\hat{i}^\dagger_{-l} + \hat{s}^\dagger_{-l}\hat{i}^\dagger_{+l}\right]|\text{vac}\rangle$$

$$|\psi_2^l\rangle = \sqrt{\frac{1}{2}}[(k_1\hat{a}^\dagger_{+l} + ik_2 b^\dagger_{-l})(ik_4\hat{a}^\dagger_{+l} + k_3 b^\dagger_{-l}) + (k_1\hat{a}^\dagger_{-l} + ik_2 b^\dagger_{+l})(ik_4\hat{a}^\dagger_{-l} + k_3 b^\dagger_{+l})]|\text{vac}\rangle$$

Measurement Operator:

$$\hat{A}_2 = |1\rangle_{a_{+l}}|1\rangle_{b_{-l}}\,{}_{a_{+l}}\langle 1|_{b_{-l}}\langle -1|$$

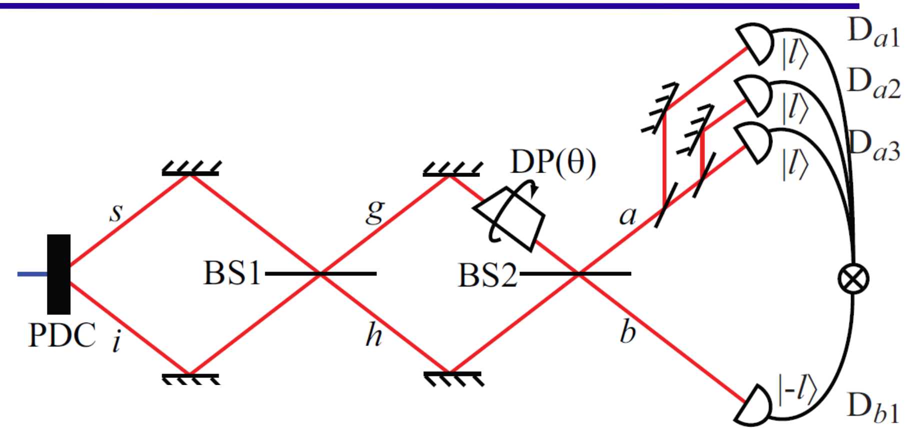
$$\langle\hat{A}_2\rangle = \text{Tr}[\hat{A}_2|\psi_2^l\rangle\langle\psi_2^l|] = \frac{1}{2}(1+\cos 4l\theta)$$

$$\Delta\theta = \frac{\langle\Delta\hat{A}_2\rangle}{|\partial\langle\hat{A}_2\rangle/\partial\theta|} = \frac{1}{4l}$$

Super-sensitive measurement with four-photon entangled state

Two-photon state:

$$|\psi_4\rangle = \sum_{l,l'} \sqrt{P_{l,l'}}|l,l'\rangle_s|-l,-l'\rangle_i,$$



$$|\psi_4^l\rangle = \frac{1}{2}\Big[|2\rangle_{s_{+l}}|2\rangle_{i_{-l}} + |2\rangle_{s_{-l}}|2\rangle_{i_{+l}} +$$

$$|1\rangle_{s_{+l}}|1\rangle_{s_{-l}}|1\rangle_{i_{+l}}|1\rangle_{i_{-l}} + |1\rangle_{s_{-l}}|1\rangle_{s_{+l}}|1\rangle_{i_{-l}}|1\rangle_{i_{+l}}\Big]$$

Measurement Operator:

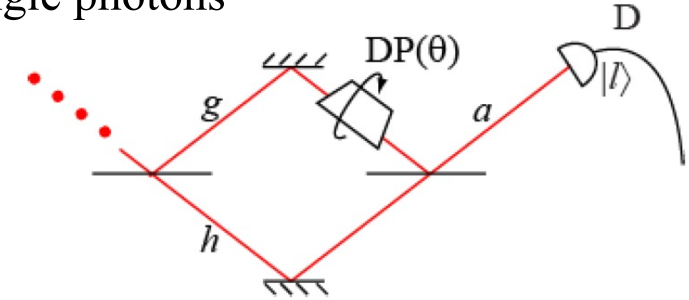
$$\hat{A}_4 = |3\rangle_{a_{+l}}|1\rangle_{b_{-l}}\,{}_{a_{+l}}\langle 3|_{b_{-l}}\langle 1|$$

$$\langle \hat{A}_4\rangle = \frac{1}{2}(1-\cos 8l\theta)$$

$$\Delta\theta = \frac{\langle \Delta \hat{A}_4\rangle}{|\partial\langle \hat{A}_4\rangle/\partial\theta|} = \frac{1}{8l}$$

Super-sensitive measurement with four-photon entangled state

(i) N-single photons

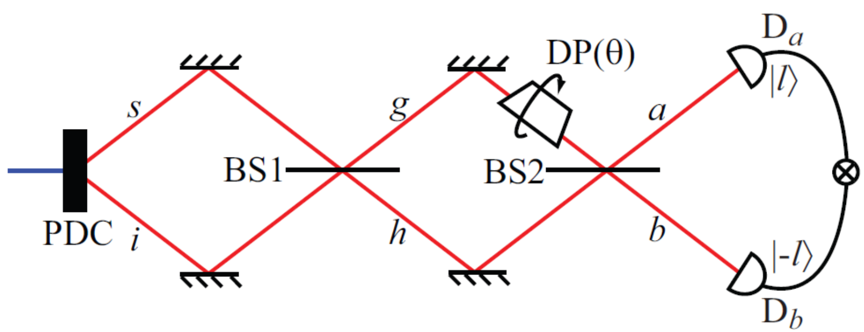


Given N photons with OAM $l\hbar$ per photon, how precisely can angular displacements be measured?

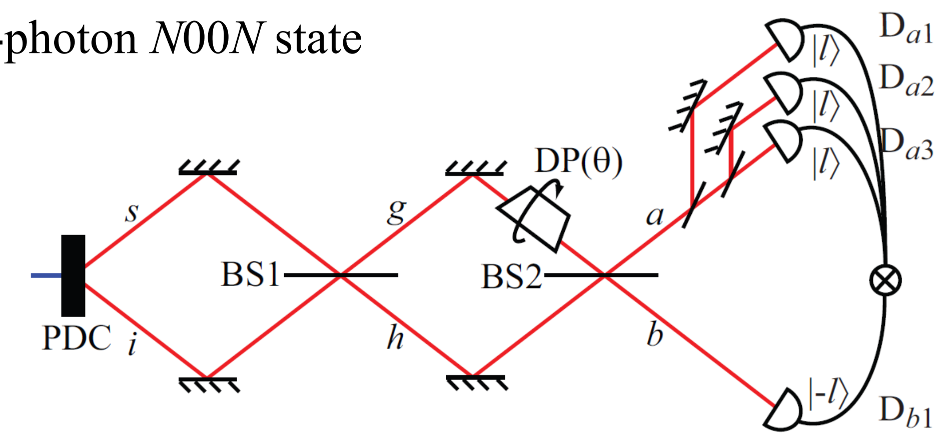
Intensity ~ $\sin 2l\theta$

Sensitivity ~ $\dfrac{\langle \Delta \hat{A}_2 \rangle}{|\partial \langle \hat{A}_2 \rangle / \partial \theta|} = \dfrac{1}{2\sqrt{N}l}$

(ii) 2-photon *N*00*N* state



(iii) 4-photon *N*00*N* state



Intensity ~ $\sin 2Nl\theta$

Sensitivity ~ $\dfrac{1}{2Nl}$

Conclusions

1. use of entangled photons having non-zero orbital angular momentum increases the resolution and sensitivity of angular displacement measurements

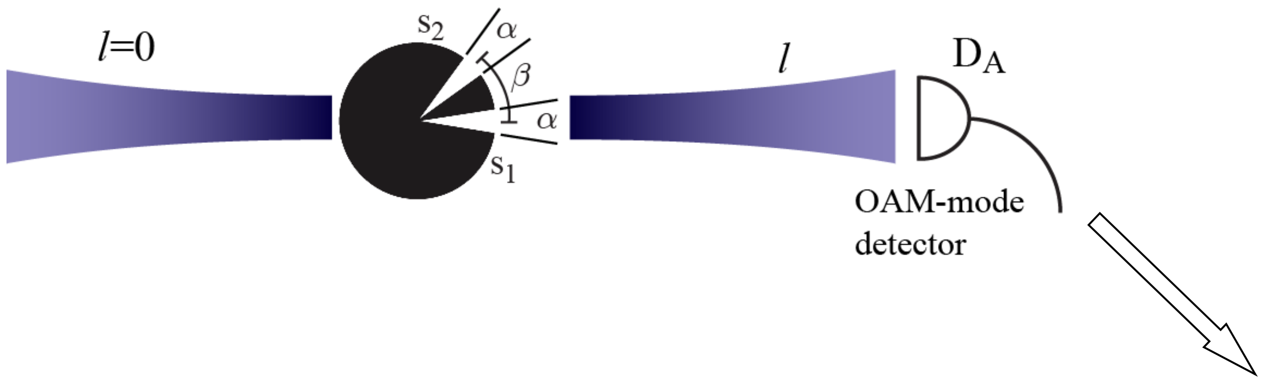
2. the resolution of improves as Nl while the sensitivity increases as $1/2Nl$.

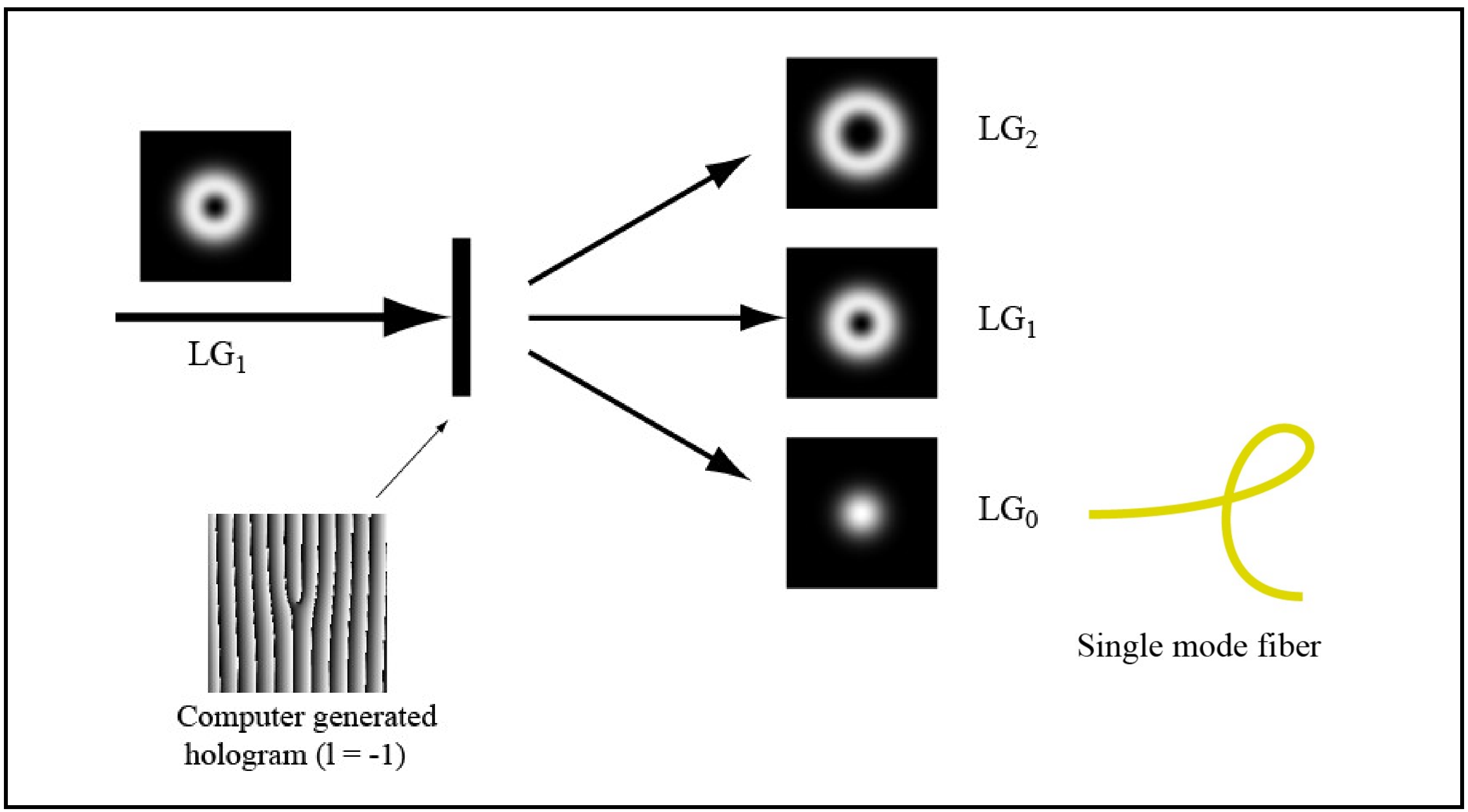
Acknowledgement

1. MURI grant from the U.S. Army Research Office

2. DARPA InPho program

Angular One-Photon Interference





Angular One-Photon Interference